Towards the Deconfinement Phase Transition in Hot Gauge Theories * †

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The phase structure of hot gauge theories with dynamical matter fields is reexamined in the canonical ensemble with respect to triality. We discuss properties of chromoelectric and chromomagnetic sectors of the theory and show whereas electric charges carrying a unit of $Z(N_c)$ charge are screened at high temperatures via dynamical matter loops, this is not the case for the $Z(N_c)$ magnetic flux. An order parameter is constructed to probe the realization of *local* $Z(N_c)$ symmetry in the magnetic sector. We argue this order parameter may be used to detect the deconfinement phase transition which is defined in terms of the screening mechanism.

Here we continue an investigation of the phase structure of hot gauge theories in the canonical ensemble with respect to triality [1]-[3]. Usually, the deconfinement phase transition is associated with the appearence of nonzero triality states in hot phase [4,5]. In particular $Z(N_c)$ gauge theories with Higgs fields have been precisely analyzed [5]. Since it was known that at zero temperature this system has two phases, a confining/screening phase and a deconfining one [6] separated by a critical line and since such a critical line was not found it has been concluded that at finite T the critical behaviour may not be present at all.

Our motivation for what follows is

I) it is not obvious a priori that "free triality states" indeed exist in deconfined phase and quite possible there might be a phase transition unrelated to the triality liberation but rather to *different screening mechanisms of triality*;

II) the previous emphasis has been put on the realization of global $Z(N_c)$ symmetry. Localizing $Z(N_c)$ looks promising [7].

We are dealing with the canonical ensemble (CE) introduced in [1,2]. This ensemble reveals the following properties [3]:

1) In the low temperature phase every state has zero triality. In the deconfined phase the whole system possesses zero triality. Since all $Z(N_c)$ noninvariant variables are projected out, the Polyakov loop (PL) itself has only little meaning in the CE and a single quark does not appear in the spectrum;

2) Metastable minima with unphysical properties are absent, all $Z(N_c)$ phases are degenerate;

3) Chiral symmetry is restored in all $Z(N_c)$ phases and at the same temperature [3].

Generalizing the A operator introduced in [7] to finite T theory, correlation function of PLs should be considered instead of Wilson loop (WL) as

$$A_t(\Sigma, R) = \frac{< L_0 L_R >_F}{< L_0 L_R >_0} \ , \tag{1}$$

where the numerator is calculated over a frustrated ensemble defined by the partition function in CE

$$Z_F = \frac{1}{N_c} \sum_{k=1}^{N_c} \int \prod_l dU_l \prod_{x,i} d\bar{\Psi}^i_x d\Psi^i_x e^{-S_F - S_q(k)} \tag{2}$$

S_q is a quark action, where $U_0 \to \exp[\frac{2\pi k}{N_c}]U_0$ for a time slice [1] and "frustrated" action is obtained from the Wilson action S_W as $S_W \to S_F = S_W(ZU_p)$. Singular transformations Z are defined on a closed dual surface Σ [3]. A similar F-ensemble may be constructed for spatial WL and the A_s operator which is the same as in zero temperature theory introduced. The A_t and A_s can be used to measure screening effects of dynamical fields and screening effects of pure gluonic interaction estimating their competition.

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If $W(C)$ is WL in $Z(N_c)$ pure gauge theory one has in the weak coupling regime

$$< W(C) > \sim \exp(-\gamma_{gl} P_C), \quad (3)$$

P_C is a perimeter of loop C. Matter fields enforce WL to decay with perimeter law at *any* coupling

$$< W(C) > \sim \exp(-\gamma_{dyn} P_C). \quad (4)$$

We refer to γ_{dyn} as coming from the dynamical screening. Keeping the system in a coupling region where the pure gauge interaction leads to an area law one has

$$< W(C) > \sim K_1 \exp(-\gamma_{dyn} P_C) + K_2 \exp(-\alpha_{gl} S).$$

α_{gl} is the string tension of pure gauge theory. In the F ensemble $K_2 \to -K_2$. It gives $A = 1$ at $C \to \infty$ and signals that dynamical screening dominates the system. There is a critical point in a pure gauge system above which

$$< W(C) > \sim K \exp(-\gamma_{dyn} P_C) + K_4 \exp(-\gamma_{gl} P_C).$$

In the F ensemble one has to change the sign of K_4. There is a competition between the kinetic and dynamical screening. One gets

$$A(\Sigma, C) = \left\{ \begin{array}{c} 1, \gamma_{dyn} < \gamma_{gl}, \\ -1, \gamma_{dyn} > \gamma_{gl}. \end{array} \right\} . \quad (5)$$

In the lower regime the kinetic screening gets stronger and $Z(N_c)$ charge can be detected. This is an *inherent* feature of the deconfined phase. In fact, (5) predicts exact equation for the critical line in the theory $\gamma_{dyn}(\alpha) = \gamma_{gl}(g^2)$ with g^2 and α the gauge and Higgs couplings, respectively.

We argue, to reveal the critical behaviour one has to analyze screening mechanisms of triality in different *gauge coupling* intervals. We *define* a deconfinement phase of the theory with dynamical matter fields as *a weak coupling phase where the screening due to gluon interactions is stronger than the dynamical one.*

At finite T the spatial WL behaves as at $T = 0$ and A_s should be a proper order parameter, too. A nontrivial value of A_s implies that a unit of $Z(N_c)$ flux is unscreened dynamically and detectable at long range.

The behaviour of A_t differs. In strong coupling regime the correlation function of PLs for pure gauge sector decays exponentially. The fermionic sector generates terms screening heavy quarks and leads to a constant value of the correlator even at spatial infinity. This implies $A_t = 1$. In weak coupling region the pure gauge sector also gives a finite value for the correlator at spatial infinity leading to a competition with the dynamical screening. Hence, the direct use of A_t as an indicator of a phase transition is impossible because both contributions are finite in the $R \to \infty$ limit. One may argue that $A_t = 1$, the stable interfaces of pure gauge system become *unstable* in the presence of dynamical matter.

Applying above idea we examine the model of $Z(2)$ gauge spins coupled to the Higgs fields at finite T. The canonical partition function of the $Z(2)$ gauge model is given by the path integral

$$Z = \frac{1}{2} \sum_{k=\pm 1} \sum_{s_l=\pm 1} \sum_{z_x=\pm 1} e^{S_W + S_H} , \quad (6)$$

$$S_W = \sum_{p_0} \lambda_0 S_{p_0} + \sum_{p_n} \lambda_n S_{p_n} , \quad (7)$$

$$S_H = S_H^{sp} + S_H^t = \sum_{x,\mu} h_\mu z_x s_\mu(x) z_{x+\mu}. \quad (8)$$

Both fields obey periodicity conditions. To calculate the operator A_t we shift the surface Σ to the Higgs part of the action getting

$$A_t(\Sigma, R) = - \frac{< L_0 L_R >_F}{< L_0 L_R >_0} , \ L_0 = \prod_{t=1}^{N_t} s(0,t). \quad (9)$$

Putting Ω as a volume enclosed by Σ we introduce

$$h_0 \to h_0(x) = \left\{ \begin{array}{c} h_0(x \notin \Omega), \\ -h_0(x \in \Omega) \end{array} \right\} . \quad (10)$$

Then if λ_0 , λ_n << 1, using strong coupling expansion we get (up to the second order)

$$< L_0 L_R > = \prod_{t=1}^{N_t} \tanh h_0(0,t) \tanh h_0(R,t)$$

$$[1 + 2 D N_t \tanh \lambda_0 \tanh^2 h_n (1 - \tanh^2 h_0)], \quad (11)$$

where D is the space dimension. Since the linking number of PL in the origin and the surface Σ is

1, it gives $A_t = 1 + o(\lambda^2)$. The expression in the square brackets is an even function of $h_0(t)$ up to the $(\tanh\lambda_0)^{L_\Sigma}$ order, where L_Σ is a linear size of domain enclosed by Σ. The corresponding plaquettes will change signs only on the boundary. Thus,

$$A_t = 1 - N_\Sigma C_1 (\tanh\lambda_0)^{L_\Sigma}, \tag{12}$$

where N_Σ is the number of frustrated plaquettes. It leads at $\Sigma \to \infty$ to the expected result $A_t = 1$.

Now if λ_0 , $\lambda_n >> 1$, it seems the fermionic contribution is suppressed as $h_0^{2N_t}$ and might be dropped relatively to the kinetic screening but it misleads. To skip this contribution, one should isolate it expanding the correlation function in small h_0 what is known to be divergent [5]. Thus, there is no direct way to separate the Debye screening from fermion screening in electric sector. In the F ensemble there is a competition between the vacuum state I with all the spins up or down depending on the triality sector and the state II when all the time-like spins are flipped inside $\Omega(\Sigma)$ relatively to links outside and the difference of classical actions gives

$$S^I - S^{II} = 2\lambda_0 N_\Sigma - 2h_0\Omega(\Sigma). \tag{13}$$

On a finite lattice there always exists a large λ_0 when the surface term wins and $A_t = -1$. When $\Sigma \to \infty$ this state can be a metastable state only as the volume term supresses the surface term in this limit. Hence, this is the state from S^{II} which dominates the thermodynamic limit and leads to $A_t = 1$.

Turning to A_s we have as above

$$A_s(\Sigma_s, C) = -\frac{< W_s(C) >_F}{< W_s(C) >_0}\ , \tag{14}$$

where $W_s(C)$ is the space-like WL defined in (6), Σ_s is a two dimensional surface on a dual lattice, Ω_s is the corresponding volume. The temporal part of the Higgs action is not affected by $Z(2)$ singular gauge transformations but for the spatial part we get in the F ensemble

$$S_H^{sp} = \sum_{x,n} h_n(x) z_x s_n(x) z_{x+n}\ , \tag{15}$$

where $h_n(x)$ is defined similarly to (10). If $\lambda_n <<$ 1 one gets as in previous case

$$A_s = 1 - N_{\Sigma_s} C_2 (\tanh\lambda)^{L_{\Sigma_s}}, \tag{16}$$

which gives $A_s = 1$ in the $\Sigma_s \to \infty$ limit. But if $\lambda_n >> 1$, the gauge part gives the following contribution to the WL

$$< W(C) > \propto \exp\left[-2P_C (e^{-2\lambda})^6\right]. \tag{17}$$

The fermionic screening is suppressed as $h_n^{P_C}$, i.e.

$$< W(C) > \propto (\tanh h_n)^{P_C}. \tag{18}$$

It allows us to expand in small h_n since there are no loops going around the lattice in space direction which could destroy the convergence. It is straightforward to calculate, e.g. $< F(\Sigma_s) >$ in leading order of small h

$$< F(\Sigma_s) >= \exp\left[-\delta N_{\Sigma_s} + O(h^6)\right], \tag{19}$$

where $\delta \approx 2h^4 \tanh\lambda$. If we shift surface Σ_s back to the pure gauge action, the dominant contribution in the F ensemble comes from configurations of gauge fields $s_n(x)$ flipped in the volume $\Omega(\Sigma_s)$ relatively to $s_n(x)$ outside of $\Omega(\Sigma_s)$. The WL changes sign in the F ensemble and we find

$$A_s(\Sigma_s, C) = -I. \tag{20}$$

The critical line in the main order is determined from $e^{-2(e^{-2\lambda^c})^6} = \tanh h_n^c$.

Thus, A_t probes directly domain walls in finite T theory and shows whether interfaces are stable. $A_s = -1$ indicates a deconfinement phase transition to a phase where the kinetic screening dominates the dynamical one.

REFERENCES

1. M. Faber, O.A. Borisenko, G.M. Zinovjev, Nucl.Phys. B444 (1995) 563.
2. M. Oleszczuk and J. Polonyi, preprint TPR 92-34, 1992.
3. M. Faber, O.A. Borisenko, G.M. Zinovjev, Nucl.Phys. B (Proc.Suppl.) 42 (1995) 484; 53 (1997) 462. Mod.Phys.Lett. A12 (1997) 949.
4. C. DeTar, L. McLerran, Phys.Lett. B119 (1982) 171.
5. H. Meyer-Ortmanns, Nucl.Phys. B230 (1984) 31.
6. R. Marra, S. Miracle Sole, Comm.Math.Phys. 67 (1978) 233.
7. J. Preskill, L.M. Krauss, Nucl.Phys. B341 (1990) 50.